Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2022

Subject Name : Linear Algebra - I

Subject Code: 4SC03LIA1 **Branch: B.Sc. (Mathematics)**

Semester: 3 **Date: 25/04/2022** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) True of False: Union of two subspace of vector space V(F) is also subspace of V(F). (01)

b) Is the set of all real numbers forms vector space over a field of complex numbers? Yes or No? (01)

c) Are vectors $v_1 = (1,0,0), v_2 = (0,2,0), v_3 = (0,0,3)$ linearly independent? Yes or No ? (01)

d) If transformation $T: R^2 \to R$ define by $T(x,y) = (x+y, x-y)$ then $T(1,2) =$ ____ (01)

(a) $(3,-1)$ (b) $(-1,3)$ (c) $(2,2)$ (d) $(-2,-2)$

e) Let T: $R^2 \to R^2$ be a linear transformation defined by $T(x,y) = (y,x)$ then T is _____. (01)

(a) One-one (b) Onto (c) Both (d) None of these

f) Let $T: R^3 \to R^3$ be a one-to-one linear transformation then the dimension of $\ker(T)$ is ? (01)

(a) 0 (b) 1 (c) 2 (d) 3

g) Define: linearly dependent and linearly independent set of vectors in vector space V. (02)

h) Define: subspace of vector space. (02)

i) Define: linear transformation from vector space V(F) to U(F). (02)

j) If $u = (1,2,3)$ and $v = (2,1,4)$ then find $< u, v >$. (02)

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Show that the vector $v = (-1,1,10)$ is a linear combination of vectors $v_1 = (1,0,1)$, $v_2 = (-2,3,-2)$, $v_3 = (-6,7,5)$. (05)

b) Show that the intersection of two subspace of vector space V is also subspace of V. (05)

c) Find cosine angle between $u = (1,2)$ and $v = (0,1)$, also verify Cauchy-Schwarz Inequality . (04)

Q-3 **Attempt all questions** **(14)**

a) Prove that a non-empty subset W of vector space $V(F)$ is subspace of $V(F)$if and only if $\alpha u + \beta v \in W \ \ \forall u, v \in W \ and \ \forall \ \alpha, \beta \in F$. (06)

b) If S is non-empty subset of vector space $V(F)$ then prove that span of S is subspace of vector space $V(F)$. (04)

c) Show that the transformation $T: R^2 \rightarrow R^3$ defined by $T(x, y) = (2x - 3y, x + 4{,}5x_2)$ is not linear transformation. (04)

Q-4 **Attempt all questions** **(14)**

a) Check whether the transformation $T: R^2 \rightarrow R^2$ defined by$T: (x, y) = (x + y, x - y)$ is linear or not? (05)

b) Check whether the set $S = \{(1,0,0), (0,1,0), (0,0,1)\}$ is a basis for R^3? (05)

c) Show that the set $S = \{v_1, v_2, v_3\}$ where $v_1 = (2,1,1), v_2 = (1,2,2)$, $v_3 = (1,1,1)$ is linearly dependent set. (04)

Q-5 **Attempt all questions** **(14)**

a) If $V$and W are two vector spaces over field F and $T: V \rightarrow W$ is linear transformation then show that$Ker(T)$ is subspace of V. (05)

b) Prove that $< u, v >= 3u_1v_1 + 2u_2v_2$ is an inner product space on $R^2$where $u = (u_1, v_1)$,$v = (v_1, v_2) \in \ R^2$. (05)

c) If $u = (u_1, u_2)$, $\ v = (v_1, v_2)$ are two vectors in R^2 then show that the R^2 is inner product space with respect to the inner product defined as $< u, v >= 4u_1v_1 + u_2v_1 + 4u_1v_2 + 4u_2v_2$. (04)

Q-6 **Attempt all questions** **(14)**

a) If $T: V \rightarrow W$ be a linear transformation then show that $Range(T)$ is subspace of W. (05)

b) Which of the following are linear transformation? (05)

$(i)T: R^2 \rightarrow R$ defined by $T(x, y) = x^2$

$(ii)\ T: R^3 \rightarrow R^2$defined by $T(x, y, z) = (2x - y + z, y - 4z)$.

c) Prove that$< u, v >= u_1v_1 - u_1v_2 + 4u_2v_1 + 4u_2v_2$is an inner product space onR^2. (04)

Q-7 **Attempt all questions** **(14)**

a) State and prove Rank-nullity theorem. (07)

b) Show that the vectors $u = (2,2,0), v = (3,0,2), w = (2,-2,2)$ formsa basis for R^3. (07)

Q-8 **Attempt all questions** **(14)**

a) let V be a vector space and $S = \{v_1, v_2, v_3, \dots \dots .. v_k\}$ be a subset of V then S is linearly dependent set if and only if one of the v_i is linear combination of other v_j in S,where $1 \leq i, j \leq k$. (07)

b) Let $S = \{v_1, v_2\}$be a subset of vector space $V(F)$ if S is linearly independent then show that $B = \{v_1 + v_2, v_1 - v_2\}$ is also linearly independent. (04)

c) Define : Inner product space. (03)